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FRACTAL GEOMETRY, FIBONACCI NUMBERS, GOLDEN RATIOS, AND PASCAL TRIANGLES AS DESIGNS

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KEYWORDS

fractal geometry, gold ratio, Fibonacci, Pascal's triangle

ABSTRACT

Fractal geometry is a part of mathematics that discusses the shape of fractals or any form that is self-similarity. A fractal can be broken down into parts that are all similar to the original fractal. Fractals have infinite detail and can have self-similar structures at different magnifications. In many cases, a fractal can be generated by repeating a pattern, which is usually in a recursive or iterative process.

In mathematics and art, two values are considered to be a golden ratio relationship if the ratio between the sum of the two values to the large value is equal to the ratio between the large value to the small value. A Fibonacci sequence is a sequence of numbers that has a unique shape. The first term of this sequence of numbers is 1, then the second term is also 1, then for the third term it is determined by adding the two previous terms so that a sequence of numbers with a certain pattern is obtained. Pascal's triangle is a geometric rule of binomial coefficients in a triangle.

Shapes that resemble fractal geometry, golden ratios and Fibonacci numbers are found in many places in this realm, for example the shapes of various plants, animals, as well as in humans themselves, even the universe. This fractal geometry can also be used to design batik motif creations, architectural design or musical art.

This paper describes how a fractal object can be made with geometric transformations that can be used to develop batik motif creations. And also look at the relationship between fractal geometry, the golden ratio, Fibonacci numbers, and Pascal's triangles and the shapes of these shapes that exist in this nature.

1. INTRODUCTION

Fractal geometry is a study in mathematics that discusses the shape of fractals or any form that is self-similarity. From several definitions of fractals, it is understood that fractals are a study in mathematics that studies shapes or geometry in which they show an infinite process of repetition. The multiplied geometry has a resemblance to each other's shapes (self-similarity), and in the arrangement of the multiplication is not bound by an orientation rule, and even tends to twist and twist with various sizes ranging from small to large. These fractals are found in nature, such as in the patterns found on the leaves and twigs of trees, in broccoli vegetables, in clusters of white clouds, in the ripples of the waves, in the details that we can see on snowflakes, and many more when we try to pay close attention to our surroundings.

Fractal geometry is a branch of mathematics that studies the properties and behavior of fractals. The word 'fractal' was first coined by Mandelbrot in 1975. Fractal geometry is a study in mathematics that discusses the shape of fractals or any form that is self-similarity. From several definitions of fractals, it is understood that fractals are a study in mathematics that studies shapes or geometry in which they show an infinite process of repetition. The multiplied geometry has a resemblance to each other's shapes (self-similarity), and in the arrangement of the multiplication is not bound by an orientation rule, and even tends to twist and twist with various sizes ranging from small to large.

2. METHOD

The research in this study adopts a theoretical approach, delving deeply into the fundamental theories related to fractal geometry, Fibonacci numbers, the golden ratio, and Pascal's triangle. Fractal geometry is defined as a branch of mathematics that studies shapes possessing self-similarity and infinite repetition processes. Foundational references are drawn from the works of Falconer (1992) and Mandelbrot (1975), along with additional sources that support these concepts.

The study employs fractal geometry models to develop designs in batik patterns, architectural design, and musical applications. Methods such as the Iterated Function System (IFS) and the Mandelbrot set are utilized in the creation of these designs. The designs and patterns are crafted using computer software that supports fractal motif creation, such as jbatik, among other relevant tools. This methodology is illustrated through case studies, including the development of batik Ulos, Gajah Oling Banyuwangi batik, and Labako batik, building on previous research findings.

Data collection is carried out through an extensive review of scientific literature, the analysis of batik artworks, and direct observation of geometric patterns found in nature. The data is gathered through literature studies and analysis of natural geometric patterns, such as leaf shapes, wave patterns, and other fractal patterns observed in nature.

The collected data is analyzed using both quantitative and qualitative approaches to identify self-similarity patterns and repetitions present in the study objects. The analysis aims to determine the relevance of fractal geometry, the golden ratio, and Fibonacci numbers in the development of designs. Geometric and statistical analysis techniques are employed to evaluate the effectiveness of fractal geometry in batik design, architecture, and music.

The implementation is then carried out by applying the results of the fractal models to the design of batik patterns, architecture, and musical compositions. Evaluation is based on the aesthetic appeal and functional value of the resulting designs. The evaluation parameters include aesthetics, complexity, and resemblance to existing natural forms. This methodology seeks to elucidate how theoretical and practical approaches are applied in research that connects fractal geometry with batik design, architecture, and musical art, demonstrating how these methods can produce works with high artistic and intellectual value.

3. RESULT AND DISCUSSION

The Fractal Set according to Falconer (1992) has 5 characters, namely:

1. It is a delicate structure, no matter how much it is enlarged
2. Too irregular, if described in ordinary geometric language
3. Have self-similarity, perhaps in terms of approach or statistically
4. The fractal dimensions are usually larger

than the topological dimensions
5. Generally it can be defined simply, perhaps recursively

In general, there are 2 types of fractal properties, namely:
1. Self-similarity
A fractal is an object that has a resemblance to itself but on a different scale, this means that a fractal object is made up of parts that have the same properties as the object. Each part of the object, when enlarged, will be identical to the object
2. Dimension
A fractal is an object that has the dimension of a real number. To compare the size of fractals, fractal dimensions are required. The fractal dimension is defined as the density of fractals occupying a metric space. The length of a line segment (two-dimensional) can be determined by measuring the length between two points. However, fractal objects cannot be measured in length, because they have infinite variations

According to the nature of similarity, there are two types of fractals, namely regular fractals and random fractals.
1. Regular fractals have exactly self-similarity, i.e. part of a fractal object resembles exactly the shape of the object as a whole when viewed from various scales.
Examples: fern leaf structure, Sierpinski triangle, Cantor set.
2. Random fractals have statistically self-similarity in that each part of the fractal object does not exactly resemble the shape of the object as a whole.
Example: Julia, Mandelbrot

The images below are fractals formed from a specific shape, obtained from various sources.

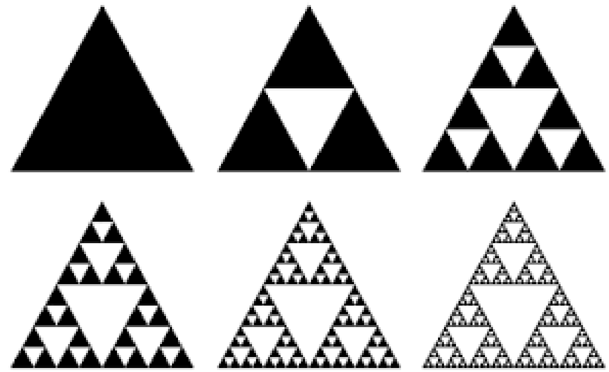


Figure 1. Sierpinski Triangle

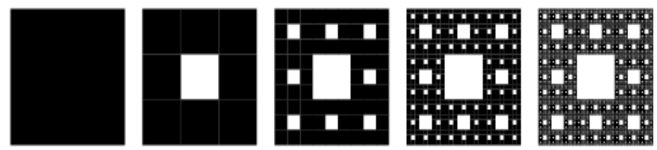


Figure 2. Sierpinski Carpet 1

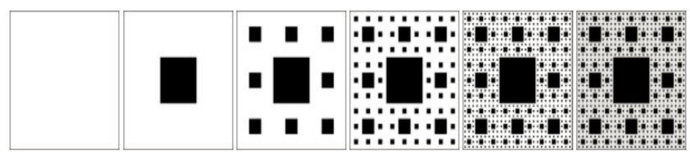


Figure 3. Sierpinski Carpet 2

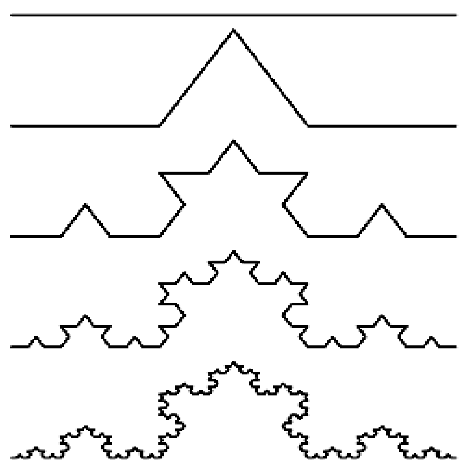


Figure 4. Kurva von Koch 1

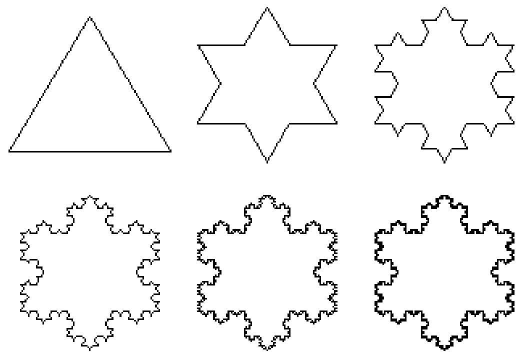


Figure 5. Kurva von Koch 2

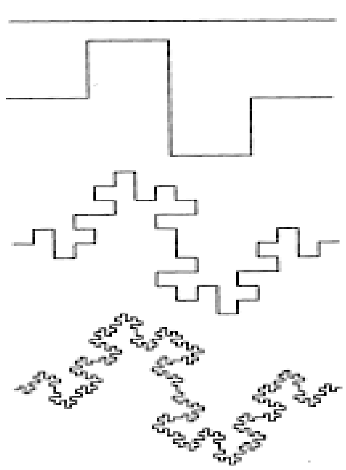


Figure 6. Minkowski

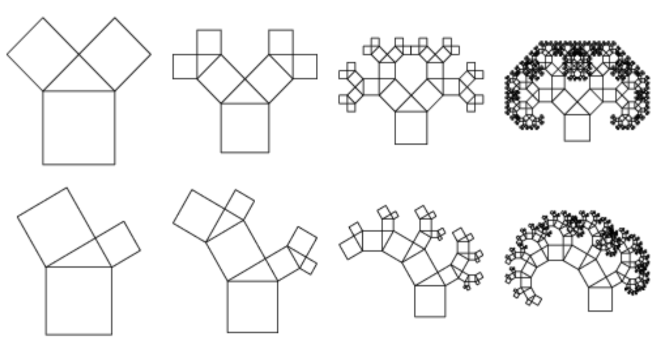


Figure 7. Pithagoras Tree

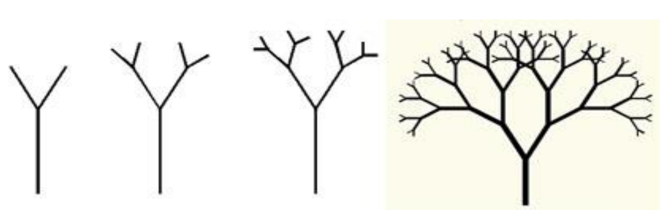


Figure 8. Tree 1

Figure 9. Tree 2

Figure 10. Tree 3

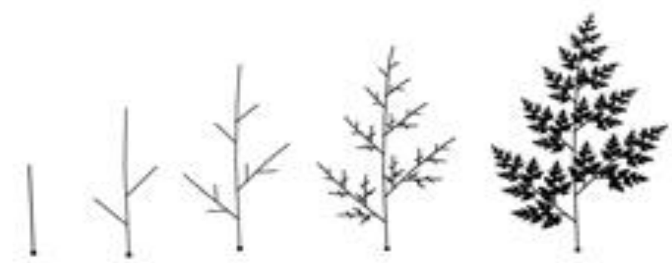


Figure 11. Tree 4

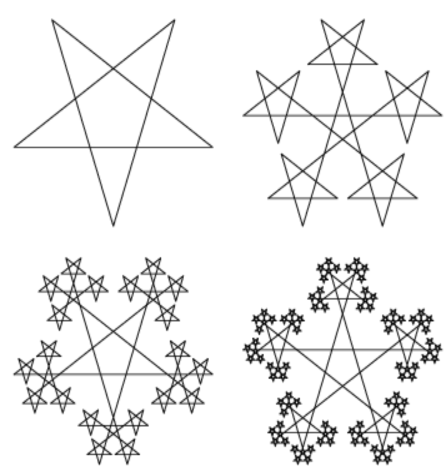


Figure 12. Star Fractal

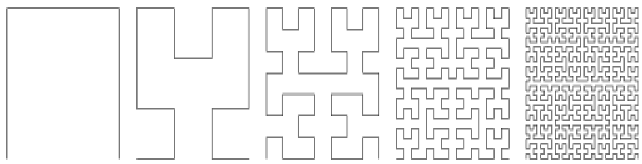


Figure 13. Hilbert Fractal 1

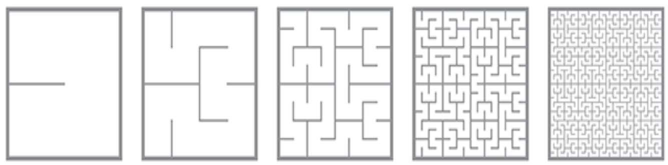


Figure 14. Hilbert Fractal 2

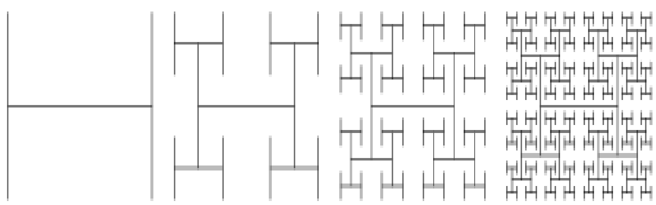


Figure 15. H Fractal

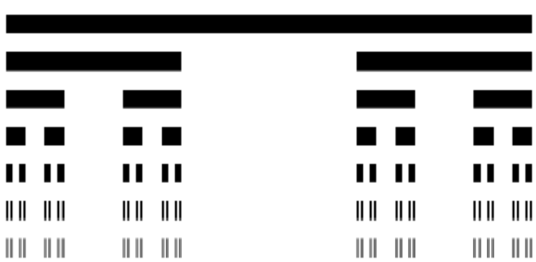


Figure 16. Cantor

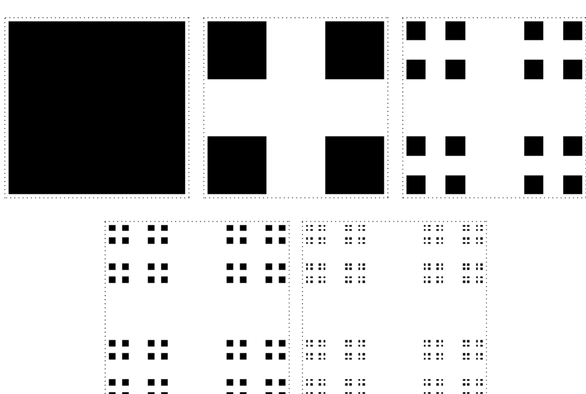


Figure 17. Dust Cantor

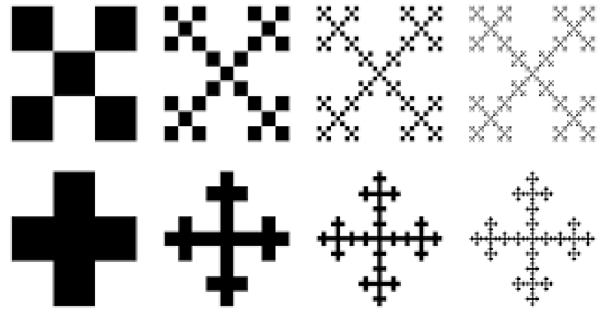


Figure 18. Box Fractal

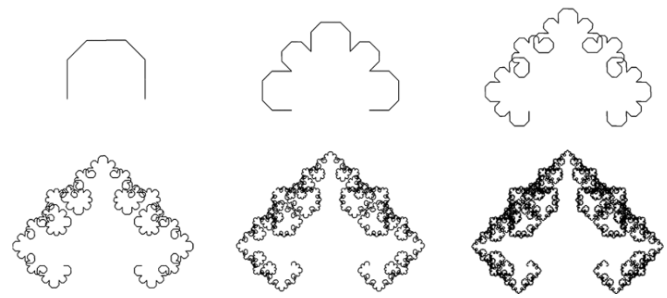


Figure 19. Levy Fractal

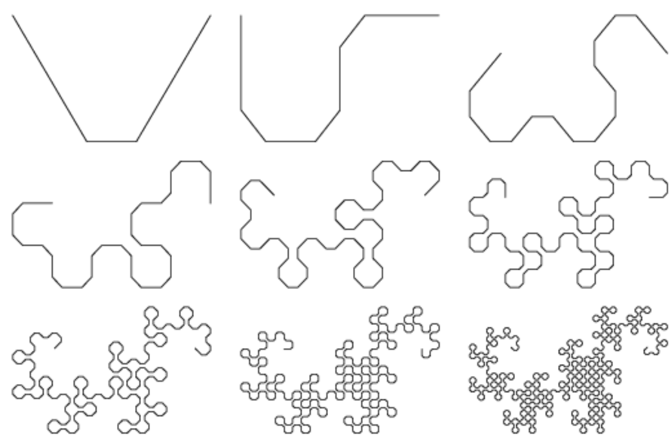


Figure 20. Dragon Curve

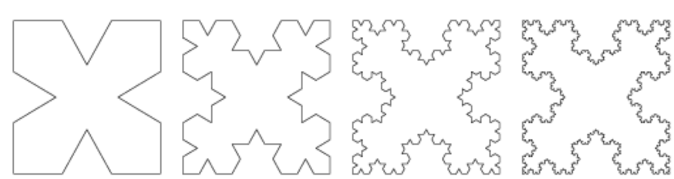


Figure 21. Cesaro Fractal

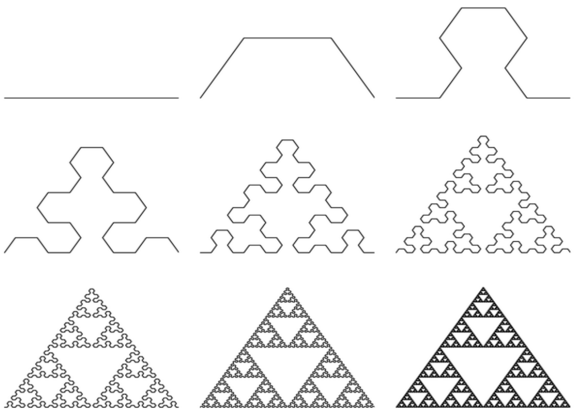


Figure 22. Sierpinski

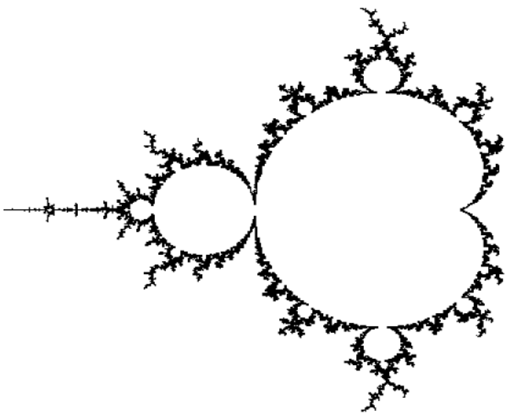


Figure 23. Mandelbrot

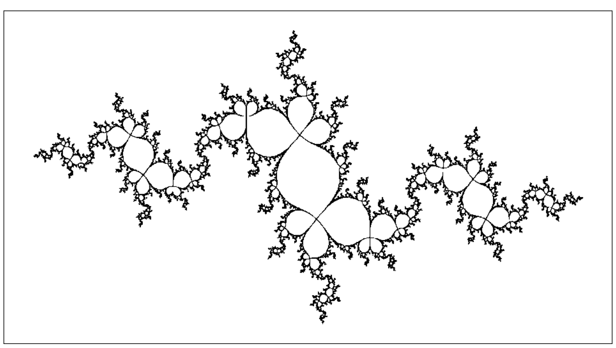


Figure 26. Julia

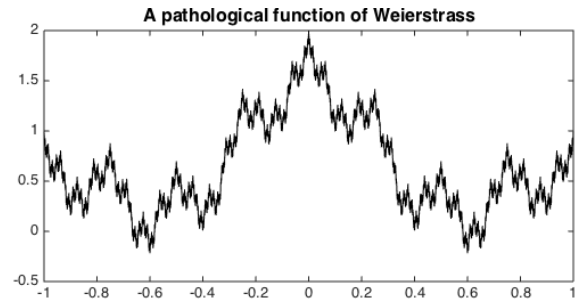


Figure 27. Weierstrass Function

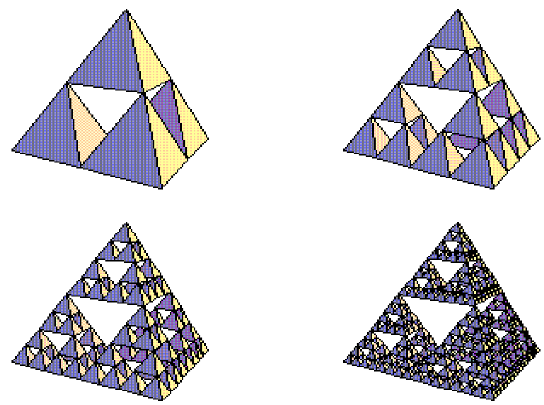


Figure 28. Tetrahedron Sierpinski

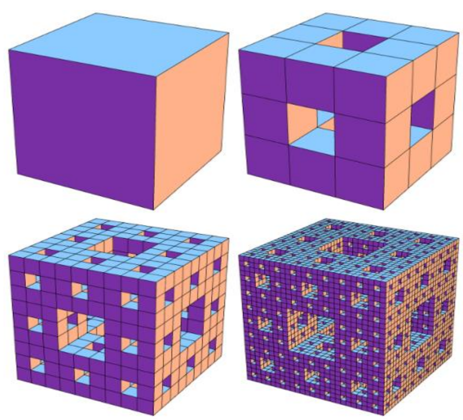


Figure 29. Menger

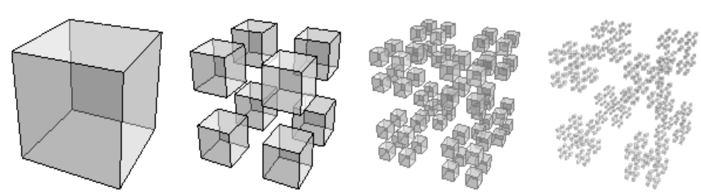


Figure 30. Dust Cantor

Fractal Batik

Batik is one of Indonesia's cultural heritage that should be preserved. The preservation of batik is carried out by the development of batik which starts from a certain processing process and then uses stamps so that the terms written batik and stamp batik appear. In addition to the batik process, developments also occur in batik motifs and patterns

There are several studies that develop batik motifs using the concept of fractals. Among them are those carried out by:

Ngarap Im Manik and Manal conducted a paper entitled "The Use of Fractal Models for the Development of Ulos Motifs". Ulos in a general sense is a traditional Batak woven fabric consisting of various types, patterns and motifs, functions and sizes. This study uses various models to form a fractal including the L-system, and the Mandelbrot Set.L-system is part of the Iterated Function System method, which is the form of a loop of a pattern with a certain size, angle and distance. The ulos motif used with the L-system method can be produced in an unlimited number, because the manufacturing process in the L-system does not have any restrictions on the symbols used. However, in fractal ulos this is limited to the use of fifteen symbols only. This application can help increase the creativity of ulos makers in choosing and designing ulos motifs so that they can produce a variety of new motifs.

Rahmatillah Agustina Meutia Dewi, Rani Rizkin Dari, and Elita Indriani conducted a research entitled "Fractal Geometry for the Re-design of Banyuwangi Gajah Oling Batik Motifs". One of the motifs on batik in Banyuwangi is the Gajah Oling Banyuwangi batik motif. So far, the process of making Gajah Oling Banyuwangi batik is still limited to written batik and stamp batik that still uses one Gajah Oling pattern. By applying fractals, batik motifs can have more than one pattern. The development of multi-pattern motifs in the Banyuwangi Elephant Oling batik motif with the application of fractals is expected to add to the beauty of the batik art itself so as to increase the richness of the batik motif and increase the selling value of batik.

Ahmad Baihaqi Y conducted a research entitled "Combining Fractal Geometry with Labako Batik". Indonesia has many batik motifs that are characteristic of various regions, one of which is Labako batik in Jember Regency. Labako batik has a characteristic of tobacco leaves as its motif, besides that there are other motifs such as coffee and leaves of plantation commodities in Jember. Along with the development, batik can be developed using fractals. Labako uses geometric transformations to obtain interesting and varied batik motifs.

Picture. Merger of Labako batik with Julia Set

Ilham Ary Wahyudie and Zanu Saputra conducted a research entitled "Redesign of Bangka Cual Woven Fabric Motif Patterns Using the Fractal Method". One of the relics of Monday's work on Bangka Island is cual woven fabric. Cual weaving was originally a traditional fabric for the nobility at the western end of Bangka Island. The use of the fractal method is very possible to redesign woven fabric motifs into batik motifs. The fractal method can help in the development of design ideas.

Eka Susanti conducted a research entitled "Variations of Palembang Batik Motifs Using the Teriteration Function System and Julia Sets". Palembang is famous for its songket fabric, but not only that, Palembang also has a distinctive batik motif. Palembang looks brighter with bright colors and still maintains traditional Palembang motifs.Some of Palembang's batik include batik with songket motifs, batik with wood grain motifs, batik with blongsong motifs, batik with rainbow jumputan motifs, lasem motifs and bungag tea motifs. With the help of a computerized system, mathematics can be visualized into an object of artistic value through batik motifs.

1. There are several software that can be used to design batik with a fractal concept, for example jbatik

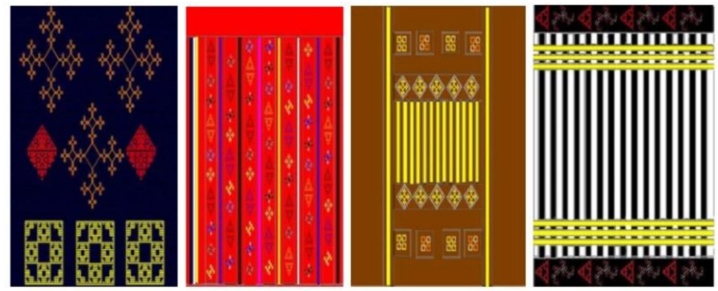


Figure 31. Motif Ulos

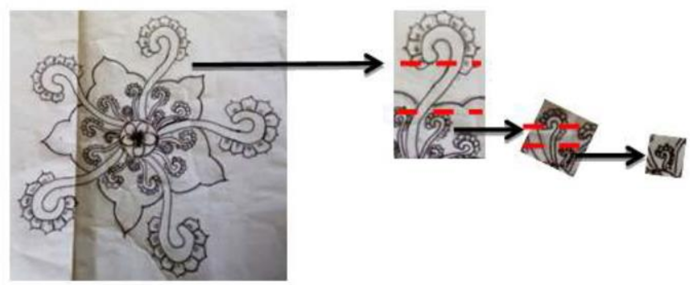


Figure 32. Oling Elephant Batik

Architectural Design

1. Architectural design can also follow the fractal concept

Sirly Intan Sayekti, Chairil Budiarto Amiuza, and Nurachmad Sujudwijono conducted a research entitled "Fractal Geometry in Singosari Temple as a Design Concept of the Singosari Archaeological Museum". Singosari Temple is one of the archipelago's architectural riches that has local characters. The architecture of the temple takes natural forms into its architecture.

The looping of natural geometry in the temple uses the vastu purusa mandala calculation rule using rectangular geometry, repeated with various large and small sizes to form a fractal geometric pattern, so that the study can be developed in architectural design. Based on the problem of conservation of past relics that need to be protected and maintained so that they can be preserved, in this study a study of the fractal geometry of the temple was carried out.

Stenly Hasang and Surijadi Supardjo conducted a research entitled "Fractal Geometry in Architectural Design". This paper explores the basic concepts of fractal geometry which is a branch of mathematics that studies the form and behavior of fractals and then applies them within the scope of architecture. The existence of fractal geometry shows that mathematics is not only a subject that always discusses counting, but can also be associated with art to produce beautiful architectural works and have high intellectual value.

Johansen Mandey, Juddy Waani, and Sangkertadi conducted a research entitled "The Application of Fractals in Apartment Architectural Design". Carl Bovill argued that the use of Euclidean geometric forms (squares, triangles, circles) resulted in flat and unnatural architectural works, while the use of fractal geometry was considered closer to the shape and transformation process that occurs in nature. The concept and method of architectural fractal design is a design process that is carried out with empirical thinking to produce an apartment architectural design.

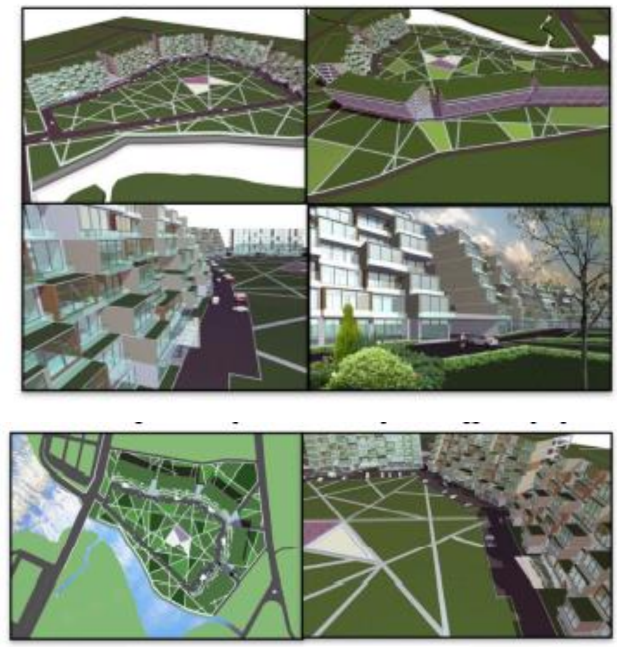


Figure 35. Apartment Perspective

- Xiaoshu – Architecture Design
- Sirly – Singosari Temple
- Stanley Hasang – architectural design
- Alexey – museum designer

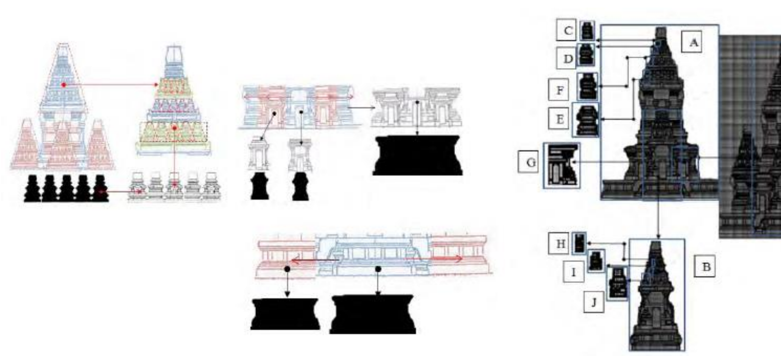


Figure 36. Singosari Temple

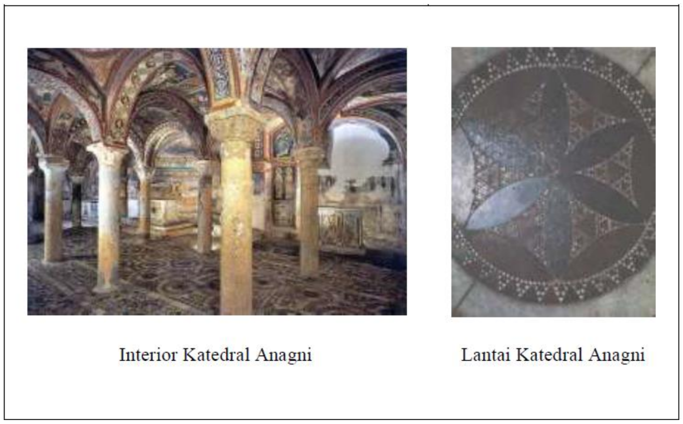


Figure 37. Interior of Anagni Cathedral

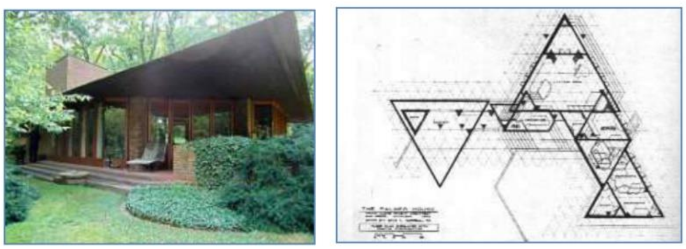


Figure 38. Palmer House

Fractals in Music

Dmitri Kartofelev and Juri Engelbrecht conducted a study entitled "Algorithmic melody composition based on fractal geometry of music". All components of that formed by music (melody, harmony, rhythm, sound) contain fractal geometry.

Fractals in nature

These fractals are found in nature, such as in the patterns found on the leaves and twigs of trees, in broccoli vegetables, in clusters of white clouds, in the ripples of the waves, in the details that we can see on snowflakes, and many more when we try to pay close attention to our surroundings. The fractal geometry of music can be exploited for the purpose of music algorithm composition.

Figure 39. Fern leaves

Figure 40.

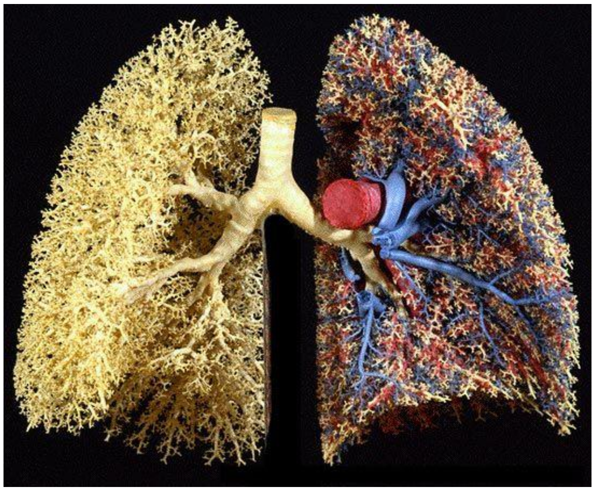


Figure 41. Human organs

Figure 42. Hosts

Figure 43. Broccoli

Fibonacci numbers

Numbers cannot be separated from mathematics. There are many types of numbers that are often found in mathematics. For example, real numbers, rational numbers, number numbers, natural numbers, complex numbers and so on. These numbers can form a pattern of number sequences and number series, be it a geometric series or an arithmetic series. The Fibonacci sequence of numbers was invented by an Italian mathematician, Leonardo Fibonacci around 1170 in his work "Liber Abaci" In "Liber Abaci" there is a problem with the reproductive problem of the Abaci rabbit. The background of the appearance of the Fibonacci line is to describe the growth of a pair of rabbits over the course of a year. Suppose the growth of the number of rabbits follows the following rule: a pair of rabbits (one female and one male) become adults within two months, and each subsequent month gives birth to a pair of rabbit cubs, male and female. If no rabbits die, how does the number of rabbit pairs develop at the beginning of each month?

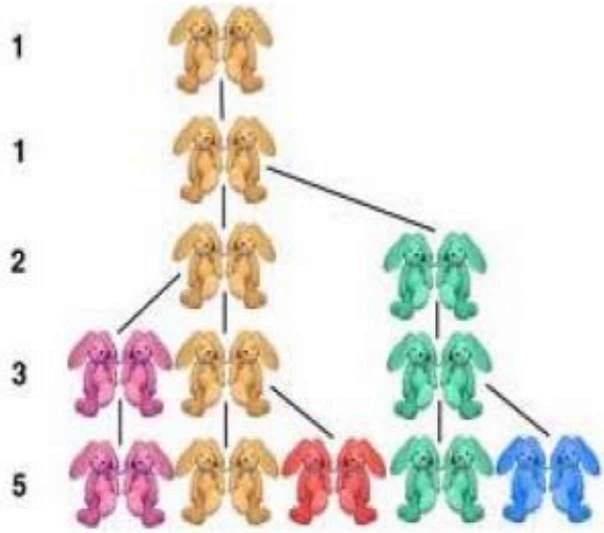


Figure 44. Reproduction of a pair of Abaci rabbits

In the first month and keuda there is a pair of rabbits. In the third month it increases by one to three pairs of rabbits. In the fourth month, two pairs of rabbits give birth until they become five pairs of rabbits, and so on. The number of rabbit pairs at the beginning of each consecutive month can be seen in the row below:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

The line above is called the Fibonacci line

Some natural phenomena are known to be representations of the Fibonacci lineup. For example, some types of flowers are as follows.

Figure 45. Fibonacci Flowers

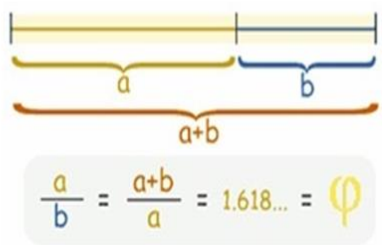
Golden Ratio

The gold ratio is 1.618205... or in the rounding number 1.618 is a convergent ratio value obtained when the terms above twelve in the Fibonacci sequence are divided by the previous quarter. In the Fibonacci lineup, f12 is worth 89, f13 is worth 144, f14 is worth 233, and f15 is worth 377. If the calculation is carried out by dividing a term in the Fibonacci series by the previous term, a number will be obtained that leads to the golden ratio, which is 1.618.

Table 1. Comparison of Fibonacci numbers

Bilangan Fibonacci	Perbandingan $\frac{\text{Bilangan ke - } (i+1)}{\text{Bilangan ke - } i}$
1	-
1	1/1 = 1,000
2	2/1 = 2,000
3	3/2 = 1,500
5	5/3 = 1,667
8	8/5 = 1,600
dst	dst

The mathematician Euclid gave the first written definition of what is referred to as the golden ratio. According to Euclid: a line is said to have been cut in an extreme and average ratio when the length of the entire line relative to the long segment is equal to the long segment compared to the short segment. Euclid explained how to cut a line in what he called the ratio of extremes and averages that became familiar with the golden ratio.



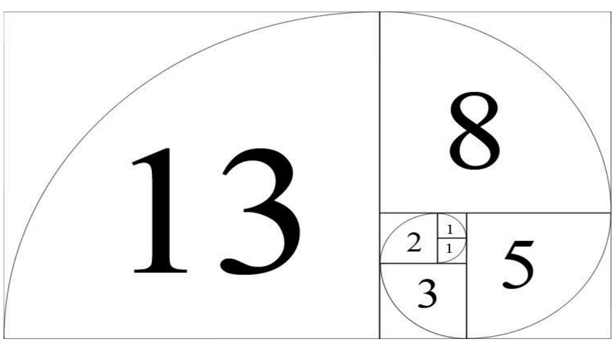


Figure 46. Golden Ratio Spiral

In the human hand, it is believed that the ratio of length between the tip of the hand to the elbow and the elbow to the base of the hand results in the golden ratio. Likewise, the ratio of dividing the top length of the palm to elbow with the tip of the palm to the base of the palm also produces the golden ratio.

Pascal's Triangle

In 1963 Blaise Pascal published a book entitled Ttrate du Triangle Arithmetique and in it there was an arrangement of numbers that became known as the Pascal triangle. Pascal's triangles are binomial coefficients or quadratic algebraic forms arranged in the form of triangles. Binomial coefficients can be expressed using combinations and algebra. By combination, binomials express many ways to create a subset with r elements of a set with n elements.

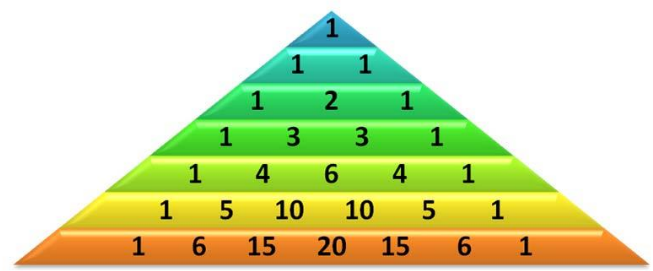


Figure 47. Pascal's Triangle

From the following figure, you can see the relationship between Fibonacci numbers and Pascal's triangles, when a diagonal straight line is made, and the numbers that the line passes through are summed up to become Fibonacci numbers.

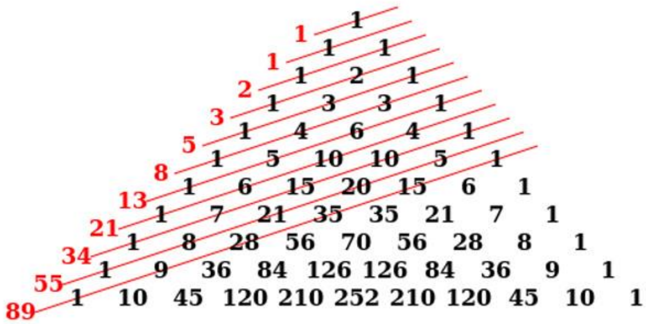


Figure 48. Fibonacci Numbers and Pascal's Triangle

4. CONCLUSION

Fractals are increasingly recognized as fundamental archetypes within the universe, serving as intricate and repeating patterns that are prevalent in various natural and artificial forms. The concept of fractals, with their self-similarity and infinite complexity, provides a foundational pattern that can be harnessed in the development of creative expressions such as batik designs, architectural structures, and musical compositions. The golden ratio and Fibonacci patterns, both closely related to fractals, are also abundant in nature, offering numerous real-world examples where these mathematical principles manifest, from the spirals of galaxies to the arrangement of leaves and the branching of trees. These natural patterns inspire and inform artistic and design practices, bridging the gap between the natural world and human creativity.

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